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Matrix theory of type IIB plane wave from membranes

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Abstract: We write down a maximally supersymmetric one parameter deformation of the field theory action of Bagger and Lambert. We show that this theory on $R \times T^2$ is invariant under the superalgebra of the maximally supersymmetric Type IIB plane wave. It is argued that this theory holographically describes the Type IIB plane wave in the discrete light-cone quantization (DLCQ).

Keywords: Gauge-gravity correspondence, M(atrix) Theories.

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Contents

1. Introduction and discussion

The supersymmetric worldvolume theory of a single M2-brane in an arbitrary eleven dimensional supergravity background was found twenty years ago [1]. The realization that branes in eleven dimensional supergravity are related by string dualities to D-branes [2] and that the low energy effective field theory of coincident D-branes is described by non-abelian super Yang-Mills theory [3], naturally prompts the search for the worldvolume theory of coincident M2-branes.

Recently, Bagger and Lambert have made an explicit proposal [4] for the Lagrangian description of the low energy limit of a stack of coincident M2-branes (see also the work of Gustavsson [5]). This work — which builds upon their prior paper [6] — incorporates the realization in [7] that the theory on coincident M2-branes should have generalized fuzzy-funnel configurations [8] described by generalized Nahm equations, and the observation in [9] that the putative gauge field of the theory should have a Chern-Simons like action.

In this paper we construct a one parameter deformation of the Bagger-Lambert theory [4] which is maximally supersymmetric. We add to their Lagrangian a mass term for all the eight scalars and fermions[1]

$$\mathcal{L}_{\rm mass} = -\tfrac{1}{2}\mu^2\,\mathrm{Tr}\left(X^I, X^I\right) + \tfrac{i}{2}\mu\,\mathrm{Tr}\left(\bar\Psi\Gamma_{3456}, \Psi\right), \tag{1.1}$$

and a Myers-like [11] flux-inducing $\mathrm{SO}(4)\times\mathrm{SO}(4)$ invariant potential[2] for the scalars

$$\mathcal{L}_{\rm flux} = -\frac{1}{6}\mu\,\varepsilon^{ABCD}\mathrm{Tr}([X^A, X^B, X^C], X^D) - \frac{1}{6}\mu\,\varepsilon^{A'B'C'D'}\mathrm{Tr}([X^{A'}, X^{B'}, X^{C'}], X^{D'}) \tag{1.2}$$

[1] The deformation of the theory on multiple $M2$-branes was first considered by Bena [10].
[2] See also [12, 13].

and show that the theory is supersymmetric. The possibility of adding the scalar mass term and the potential term for four of the scalars was considered in [14]. Here we show that if we give a mass to all the scalars and fermions and turn on the potential (1.2) for all the scalars that we can find a deformation of the supersymmetry transformations of the Bagger-Lambert theory [4] in such a way that the deformed field theory remains fully supersymmetric. This construction yields a novel maximally supersymmetric, Poincare invariant three dimensional field theory.

We further argue that the deformed field theory compactified on $R \times T^2$ provides the Matrix theory [15] description[3] of Type IIB string theory on the maximally supersymmetric plane wave[4] [20]. We show that the deformed field theory on $R \times T^2$ has as its algebra of symmetries the superisometry algebra of the Type IIB plane wave, as expected from a holographic dual theory. The deformed field theory on $R \times T^2$ is proposed as the nonperturbative formulation of the Type IIB string theory in the discrete lightcone quantization (DLCQ).

We show that the supersymmetric ground states of the deformed theory are given by a discrete set of states that have an interpretation[5] as a collection of fuzzy S^3's [14], where

$$[X^A, X^B, X^C] = -\mu\epsilon^{ABCD} X^D, \qquad X^{A'} = 0 \tag{1.3}$$

or alternatively:

$$[X^{A'}, X^{B'}, X^{C'}] = -\mu\epsilon^{A'B'C'D'} X^{D'}, \qquad X^A = 0. \tag{1.4}$$

We identify these states of the deformed theory with the states in the Hilbert space of the Type IIB plane wave with zero light-cone energy, which correspond to configurations of D3-brane giant gravitons in the Type IIB plane wave background [16] with fixed longitudinal momentum.

It would be interesting to use the deformed field theory we write down to capture stringy physics in the Type IIB plane wave. In the regime when the bulk Type IIB string theory is weakly coupled the deformed field theory can be dimensionally reduced to 1+1 dimensions

and it would be interesting to extract the Type IIB lightcone string field theory interaction vertices in the plane wave background from the reduced theory.

The Lagrangian of the deformed theory is based on the same 3-algebra structure of [4] (we review it in section 2). Even though the construction of Bagger-Lambert and in this paper certainly provide new constructions of supersymmetric field theories, the precise connection with the worldvolume physics of coincident M2-branes still remains to be understood. Currently a single 3-algebra structure is known even though several constructions

[3] In [16] (see also [17]), an analogous deformation of the D0 brane Lagrangian was proposed as the Matrix theory description of the maximally supersymmetry plane wave of eleven dimensional supergravity.

[4] Or given the interpretation in [18, 19] for the known 3-algebra $\mathcal{A}_4$, possibly an orientifold projection of the maximally supersymmetric plane wave background.

[5] These states have yet another space-time interpretation as M2-branes polarizing in the presence of flux into M5-branes with S^3 topology. The supergravity description of these ground states of the deformed theory were found in [21] (see also [22]).

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have recently been considered [23, 24], and for the known case there are subtleties identifying the M2 content of the theory as well as and the spacetime geometry in which the M2 branes are embedded [14, 23, 25, 18, 19]. The possibility of relaxing the conditions on the 3-algebra to construct new examples have been considered [26, 27] and the addition of a boundary to the theory has been considered in [28].

Establishing in more detail the M2 brane interpretation of our deformed theory is important in understanding the deformed field theory found in this paper as the Matrix theory description of the Type IIB plane wave.

The plan of the rest of the paper is as follows. In section 2 we quickly review the Bagger-Lambert theory [14] and introduce the deformation of the Lagrangian and the supersymmetry transformations that gives rise to a new maximally supersymmetric Lagrangian in three dimensions. In section 3 we argue that the deformed theory on $R \times T^2$ provides the Matrix theory description of the maximally supersymmetric Type IIB plane wave. We show that the theory on $R \times T^2$ has precisely the same symmetry algebra as the Type IIB plane wave and identify the supersymmetric grounds states of the deformed theory with the states in the Type IIB plane wave with zero light-cone energy, which correspond to configurations of D3-brane giant gravitons. In appendix A we present some details of the calculation of the deformed supersymmetry transformations while in appendix B we write down the Noether charges of the deformed field theory on $R \times T^2$ and show that they satisfy the Type IIB plane wave superalgebra.

2. Deformed supersymmetric field theory

In [4], a new maximally supersymmetric Lagrangian in three dimensions has been found. The authors have proposed that this theory describes the low energy dynamics[6] of a stack of M2-branes. It encodes the interactions of the eight scalar fields X^I transverse to the M2-branes, the worldvolume fermions Ψ and a non-propagating gauge field A_μ. The Lagrangian is given by [4]

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}(D_\mu X^{aI})(D^\mu X^I_a) + \frac{i}{2}\bar{\Psi}^a\Gamma^\mu D_\mu \Psi_a + \frac{i}{4}\bar{\Psi}_b \Gamma_{IJ} X^I_c X^J_d \Psi_a f^{abcd} + \\ & -V + \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb} A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right),\end{aligned} \tag{2.1}$$

where V is the scalar potential

$$V = \tfrac{1}{12} f^{abcd} f^{efg}{}_d X^I_a X^J_b X^K_c X^I_e X^J_f X^K_g \equiv \frac{1}{2\cdot 3!}\mathrm{Tr}([X^I, X^J, X^K][X^I, X^J, X^K]) \tag{2.2}$$

and the covariant derivative of a field Φ is given by

$$D_\mu \Phi^a = \partial_\mu \Phi^a - \tilde{A}^a_{\mu b}\Phi^b, \tag{2.3}$$

where $\tilde{A}^a_{\mu b} \equiv f^a{}_{bcd} A^{cd}_\mu$.

[6] In this $l_p \to 0$ limit, higher derivative corrections can be ignored.

This theory is based on a novel algebraic structure [4], a 3-algebra $\mathcal{A}_n$ with generators T^a — where $a = 1, \dots \dim \mathcal{A} = n$ — and on a 3-product:

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d. \tag{2.4}$$

In [4], the structure constants f^{abcd} are taken to be totally antisymmetric and to satisfy the fundamental identity

$$f^{aef}{}_g f^{bcdg} - f^{bef}{}_g f^{acdg} + f^{cef}{}_g f^{abdg} - f^{def}{}_g f^{abcg} = 0 \tag{2.5}$$

which generalizes the familiar Jacobi identity of Lie algebras. The algebra indices are contracted with a prescribed non-degenerate metric $h^{ab} = \mathrm{Tr}(T^a, T^b)$. Thus far the only examples of 3-algebras found are of the type $\mathcal{A}_4 \oplus \mathcal{A}_4 \oplus \dots \oplus \mathcal{A}_4 \oplus C_1 \oplus \dots C_l$, where $\mathcal{A}_4$ is defined by $f^{abcd} = \epsilon^{abcd}$ and C_i denote central elements in the algebra. The supersymmetric deformation we find in this paper applies to any 3-algebra with totally antisymmetric structure constants which satisfies the fundamental identity (2.5).

We now find a deformation of the action and supersymmetry transformations of the action of Bagger and Lambert [4] that is maximally supersymmetric. The new Lagrangian is given by

$$\tilde{\mathcal{L}} = \mathcal{L} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{flux}}, \tag{2.6}$$

where $\mathcal{L}$ is the Bagger-Lambert theory in (2.1) and:

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -\frac{1}{2}\mu^2 \mathrm{Tr}(X^I, X^I) + \frac{i}{2}\mu \mathrm{Tr}(\bar{\Psi}\Gamma_{3456}, \Psi) \\ \mathcal{L}_{\text{flux}} &= -\frac{1}{6}\mu \varepsilon^{ABCD} \mathrm{Tr}([X^A, X^B, X^C], X^D) - \frac{1}{6}\mu \varepsilon^{A'B'C'D'} \mathrm{Tr}([X^{A'}, X^{B'}, X^{C'}], X^{D'}). \end{aligned} \tag{2.7}$$

The transverse index has been decomposed as $I = (A, A')$ where $A = 3, 4, 5, 6$ and $A' = 7, 8, 9, 10$ and Ψ is an eleven dimension Majorana spinor satisfying the constraint $\Gamma_{012}\Psi = -\Psi$, where the Γ-matrices satisfy the Clifford algebra in eleven dimensions. This deformation of the Lagrangian is analogous to the deformation of the Lagrangian of D0-branes considered in [16]. This deformation when restricted to only four of the scalars has been considered in [14].

The deformed Lagrangian now depends on the paramater μ. The mass term $\mathcal{L}_{\text{mass}}$ gives mass to all the scalars and fermions in the theory, while $\mathcal{L}_{\text{flux}}$ has the interpretation of the scalar potential[7] generated by a background four-form flux of eleven dimensional supergravity, of the type found by Myers [11] (see also [12, 13]) in the context of D-branes in the presence of background fluxes.

The deformed theory (2.6) breaks the SO(8) R-symmetry of the undeformed theory (2.1) down to SO(4) $\times$ SO(4). The deformed theory is nevertheless invariant under

[7] We note that if we use the proposal made by Mukhi and Papageorgakis [23] to obtain by compactification the theory on D2 branes, that $\mathcal{L}_{\text{flux}}$ does indeed reduce to the known Myers term.

sixteen linearly realized supersymmetries. The supersymmetry transformations of the deformed theory are given by

$$\begin{aligned}
\tilde{\delta} X^I &= i\bar{\epsilon}\Gamma^I\Psi \\
\tilde{\delta}\Psi &= D_\mu X^I\Gamma^\mu\Gamma^I\epsilon - \frac{1}{6}[X^I, X^J, X^K]\Gamma^{IJK}\epsilon - \mu\Gamma_{3456}\Gamma^I X^I\epsilon \\
\tilde{\delta}\tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_\mu\Gamma_I X^I_c\Psi_d f^{cdb}{}_a,
\end{aligned} \tag{2.8}$$

where ϵ is a constant eleven dimensional Majorana spinor satisfying the constraint $\Gamma_{012}\epsilon = \epsilon$. By setting $\mu \to 0$ we recover the supersymmetry transformations of the undeformed theory (2.1) found by Bagger-Lambert [4]. The proof that the action (2.6) is invariant under the supersymmetry transformations is summarized in appendix A.

The deformed action (2.6) is also invariant under sixteen non-linearly realized supersymmetries if the 3-algebra $\mathcal{A}_n$ has a central element $C = T^0$, so that $f^{abc0} = 0$. Then the action (2.6) is invariant under the following non-linear supersymmetry transformations[8]

$$\begin{aligned}
\delta_n X^I_a &= 0 \\
\delta_n \Psi &= \exp\left(-\frac{\mu}{3}\Gamma_{3456}\Gamma_\mu\sigma^\mu\right) T^0\eta \\
\delta_n \tilde{A}_\mu{}^b{}_a &= 0
\end{aligned} \tag{2.9}$$

where now η is an eleven dimensional Majorana spinor satisfying the constraint $\Gamma_{012}\eta = -\eta$ and σ^μ are the three dimensional field theory coordinates.

The field theory with Lagrangian (2.6), (2.8) and with supersymmetry transformations (2.8), (2.9) defines a novel maximally supersymmetric Poincare invariant three dimensional field theory with SO(4) $\times$ SO(4) R-symmetry.

3. Deformed theory as DLCQ of type IIB plane wave

In [29, 30], the theory of coincident M2-branes on $R\times T^2$ was argued[9] to provide the Matrix theory [15] description of Type IIB string theory in flat space, extending the Matrix string theory description in [31, 32] to Type IIB string theory.

In this section we argue that the three dimensional deformed field theory (2.6) on $R \times T^2$ provides the Matrix theory[10] description of the maximally supersymmetric Type IIB plane wave background [20]:

$$\begin{aligned}
ds^2 &= 2dx^+dx^- - \mu^2 x^I x^I dx^+dx^+ + dx^I dx^I \\
F_{+1234} &= F_{+5678} = 2\mu.
\end{aligned} \tag{3.1}$$

As in the case of flat space, the modular parameter τ of the torus on which the deformed field theory is defined determines the complexified coupling constant of Type IIB string theory $\tau = C_0 + i/g_s$ [38].

[8] The Bagger-Lambert theory (2.1) is also invariant under the sixteen nonlinearly realized supersymmetries obtained by setting $\mu \to 0$.

[9] At that time there was no Lagrangian description of the coincident M2-brane theory.

[10] For a different proposal for the Matrix theory of the Type IIB plane wave see [33]. For the DLCQ description of the plane wave in terms of a sector of a quiver gauge theory see [34]. See also [35 – 37].

In this paper we have constructed a one parameter deformation of the Bagger-Lambert field theory that preserves all the thirty-two supersymmetries.

It is therefore natural to propose that the deformed theory (2.6) found in this paper is the Matrix theory description of the Type IIB plane wave. Also as $\mu \to 0$ the plane wave background (3.1) reduces to flat space just as the deformed field theory (2.6) goes over to the Bagger-Lambert theory (2.1), which as the candidate theory for multiple M2-branes is the Matrix theory for flat space.[11]

Matrix theory describes nonperturbatively a string/M-theory background in the discrete light cone quantization (DLCQ) [39]. In this quantization we consider a string/M-theory background with a compactified lightlike coordinate $x^- \simeq x^- + 2\pi R$ in a sector with quantized longitudinal momentum $P^+ = N/R$. The Matrix theory description of a string/M-theory background with some prescribed asymptopia must realize the same symmetries as those of the asymptotic background with the lightlike identification $x^- \simeq x^- + 2\pi R$.

If we consider the DLCQ of Type IIB string theory in $R^{1,9}$, then the $I\,\mathrm{SO}(1,9)$ symmetry algebra of Minkowski space is broken by the x^- identification to the centrally extended Super-Galileo algebra $SGal(1,8)$ [39], where the central extension corresponds to P^+.

The Type IIB plane wave background (3.1) is invariant under thirty-two supersymmetries and under a thirty-dimensional bosonic symmetry algebra [20]. Unlike in flat space, the $x^- \simeq x^- + 2\pi R$ does not break any of these symmetries. It is useful to gain intuition on the action of these symmetries to notice that the bosonic symmetries of the Type IIB plane wave background (3.1) can be identified with the centrally extended Newton-Hooke algebra[12] $NH(1,8)$. This algebra of symmetries is the non-relativistic contraction[13] of the isometry algebra of AdS_9, just like the $Gal(1,8)$ symmetry algebra of Matrix string theory in flat space arises in the non-relativistic contraction of the isometry algebra of $R^{1,8}$. As in the case of flat space, the central extension corresponds to P^+. Therefore the non-central generators of $NH(1,8)$ are given by H, P^I, K^I, J^{AB} and $J^{A'B'}$, which generate time translations, spatial translations, boosts and rotations respectively, and where the transverse index has been decomposed as $I = (A, A')$.

The deformed field theory (2.6) is manifestly invariant under the action of H, J^{AB} and $J^{A'B'}$, which correspond in the deformed field theory (2.6) to the Hamiltonian and the $\mathrm{SO}(4)\times\mathrm{SO}(4)$ R-symmetry charges of the three dimensional field theory. The non-manifest symmetries that remain to be realized are the translations P^I and boosts K^I. We now consider the following non-linear action of these generators on the fields of the deformed

[11] See [18, 19] for subtleties with this interpretation.

[12] This algebra has appeared previously in the context of non-relativistic symmetries of string theory in e.g. [40 – 43].

[13] The flux in (3.1) actually breaks the SO(8) rotation symmetry of the contracted algebra down to $\mathrm{SO}(4) \times \mathrm{SO}(4)$.

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field theory (2.6)

$$P^J: \quad \begin{aligned} \delta X^I &= a^J \delta^{IJ} \cos(\mu\sigma^0) T^0 \\ \delta\Psi &= 0 \\ \delta \tilde{A}_\mu{}^b{}_a &= 0 \end{aligned} \tag{3.2}$$

and

$$K^J: \quad \begin{aligned} \delta X^I &= v^J \delta^{IJ} \frac{\sin(\mu\sigma^0)}{\mu} T^0 \\ \delta\Psi &= 0 \\ \delta \tilde{A}_\mu{}^b{}_a &= 0, \end{aligned} \tag{3.3}$$

where σ^0 is the field theory time coordinate and T^0 is a central element in the 3-algebra $\mathcal{A}$. Note that in the flat space limit $\mu \to 0$ we recover the usual Galilean transformations. Under the action of the transformations (3.2) and (3.3) the deformed Lagrangian (2.6) changes by a total derivative. This provides the field theory explanation for the existence of the central extension P^+ in $NH(1,8)$, as central extensions of symmetry algebras are always associated with symmetry transformations that result in quasi-invariant Lagrangians. The central extension appears in the commutator of translations and boosts:

$$[P^I, K^J] = i\delta^{IJ} P^+. \tag{3.4}$$

The original three dimensional Poincare symmetry of the field theory is broken by compactification to $R \times T^2$ to just the translation algebra. The time translation generator H is identified with the Type IIB Hamiltonian. The translation generators along the T^2 can be identified with central charges of the superalgebra [44, 30]. These central charges are associated with fundamental strings and D1 strings wrapping the longitudinal direction of the Type IIB plane wave (3.1). The geometrical action of SL$(2,Z)$ on the T^2 on which the theory is defined exchanges the fundamental and D1 strings in the way expected from duality [38].

The supercharge generating the supersymmetry transformations (2.8) correspond to the dynamical supersymmetries of the Type IIB plane wave (3.1) while the supercharges generating the supersymmetry transformations (2.9) correspond to the kinematical supersymmetries of the plane wave. Thus combining the bosonic symmetries with the supersymmetry transformations found in the section 2 we conclude that the deformed field theory (2.6) is invariant under $SNH(1,8)$, or equivalenty under the superisometry algebra of the Type IIB plane background (3.1) in the DLCQ. In appendix B we write the Noether charges of the deformed field theory on $R \times T^2$ and show that the commutation relations are those of the Type IIB plane wave (3.1).

Therefore the deformed field theory (2.6) has the necessary ingredients to be the Matrix theory description of the Type IIB plane wave (3.1).

3.1 Deformed field theory vacua and type IIB plane wave giant gravitons

Type IIB string theory on the plane wave background (3.1) contains in its Hilbert space states with zero light-cone energy — where $H = 0$ — that preserve half of the supersymmetry [16]. They correspond to configurations of giant gravitons. A giant graviton in (3.1) is a $D3$ brane which wraps S^3 or $\tilde{S}^3$ at $x^- = 0$, where S^3 ($\tilde{S}^3$) is the sphere of the first (second) R^4 in the plane wave geometry (3.1). The radius of the giant graviton is determined by the longitudinal momentum P^+ carried by the $D3$-brane [16]:

$$\frac{L^2}{\alpha'} = 2\pi g_s \mu P^+ \alpha'. \tag{3.5}$$

When considering the DLCQ of the Type IIB plane wave, the total longitudinal momentum is quantized $P^+ = N/R$. Therefore, the $H = 0$ states of the DLCQ of the plane wave are labeled by partitions of N, and each state describes a configuration of $D3$-branes whose total longitudinal momentum is $P^+ = N/R$. These $D3$-brane configurations preserve half of the supersymmetries. More precisely, they preserve all the sixteen linearly realized supersymmetries while they break all of the non-linearly supersymmetries of the plane wave background.

The deformed field theory (2.6) also contains in its Hilbert space zero energy states that preserve half of the supersymmetries of the theory. These ground states are described by constant scalar fields satisfying

$$[X^A, X^B, X^C] = -\mu\epsilon^{ABCD} X^D, \qquad X^{A'} = 0 \tag{3.6}$$

or alternatively:

$$[X^{A'}, X^{B'}, X^{C'}] = -\mu\epsilon^{A'B'C'D'} X^{D'}, \qquad X^A = 0, \tag{3.7}$$

where we have split the transverse index $I = (A, A')$, with $A = 3, 4, 5, 6$ and $A' = 7, 8, 9, 10$. These solutions automatically satisfy the supersymmetry condition[14] $\tilde{\delta}\Psi = 0$ in (2.8) and preserve all the linearly realized supersymmetries while they break the non-linearly realized supersymmetries, just like the giant gravitons in the Type IIB plane wave (3.1). It is straightforward to show that these states also have $H = 0$.

We identify these states of the deformed field theory with the giant graviton configurations of the Type IIB plane wave. Further work on 3-algebras and their representation theory is important to further understand the Matrix theory proposal of this paper.

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[14] The supersymmetry conditions of [33] were analyzed in [45].

A. Supersymmetry of deformed field theory

We first note that the susy variation (2.8) can be decomposed as

$$\tilde{\delta} = \delta_\epsilon + \delta_\mu, \tag{A.1}$$

where δ_ϵ are given in

$$\begin{aligned} \delta_\epsilon X^I_a &= i\bar{\epsilon}\Gamma^I \Psi_a \\ \delta_\epsilon \Psi_a &= D_\mu X^I_a \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X^I_b X^J_c X^K_d f^{bcd}{}_a \Gamma^{IJK}\epsilon \\ \delta_\epsilon \tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_\mu \Gamma_I X^I_c \Psi_d f^{cdb}{}_a. \end{aligned} \tag{A.2}$$

and

$$\begin{aligned} \delta_\mu X^I_a &= 0 \\ \delta_\mu \Psi &= -\mu \Gamma_{3456}\Gamma^I X^I \epsilon \\ \delta_\mu \tilde{A}_\mu{}^b{}_a &= 0, \end{aligned} \tag{A.3}$$

where ϵ is an eleven dimensional Majorana spinor subject to the constraint $\Gamma_{012}\epsilon = \epsilon$. Since $\tilde{\mathcal{L}} = \mathcal{L} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{flux}}$, we have that:

$$\tilde{\delta}\tilde{\mathcal{L}} = \delta_\epsilon \mathcal{L} + \delta_\epsilon \mathcal{L}_{\text{mass}} + \delta_\epsilon \mathcal{L}_{\text{flux}} + \delta_\mu \mathcal{L} + \delta_\mu \mathcal{L}_{\text{mass}} + \delta_\mu \mathcal{L}_{\text{flux}}. \tag{A.4}$$

In [4] it has already been shown that $\delta_\epsilon \mathcal{L} = 0$ up to total derivatives. It is trivial to see that $\delta_\mu \mathcal{L}_{\text{flux}} = 0$. The other terms are:

$$\begin{aligned} \delta_\epsilon \mathcal{L}_{\text{mass}} &= -\mu^2 \text{Tr}(X^I, i\bar{\epsilon}\Gamma^I \Psi) + i\mu \text{Tr}(D_\mu X^I, \bar{\Psi}\Gamma_{3456}\Gamma^\mu \Gamma^I \epsilon) \\ &\quad -i\frac{1}{6}\mu \text{Tr}([X^I, X^J, X^K], \bar{\Psi}\Gamma_{3456}\Gamma^{IJK}\epsilon) \end{aligned} \tag{A.5}$$

$$\begin{aligned} \delta_\epsilon \mathcal{L}_{\text{flux}} &= i\frac{2}{3}\mu \varepsilon^{ABCD} \text{Tr}([X^A, X^B, X^C], \bar{\Psi}\Gamma^D \epsilon) \\ &\quad +i\frac{2}{3}\mu \varepsilon^{A'B'C'D'} \text{Tr}([X^{A'}, X^{B'}, X^{C'}], \bar{\Psi}\Gamma^{D'}\epsilon) \\ &= -i\frac{2}{3}\mu \text{Tr}([X^A, X^B, X^C], \bar{\Psi}\Gamma^{ABC}\Gamma_{3456}\epsilon) \\ &\quad +i\frac{2}{3}\mu \text{Tr}([X^{A'}, X^{B'}, X^{C'}], \bar{\Psi}\Gamma^{A'B'C'}\Gamma_{3456}\epsilon) \end{aligned} \tag{A.6}$$

In the last step of (A.6) we have used

$$\varepsilon^{ABCD}\Gamma^D = -\Gamma^{ABC}\Gamma_{3456}, \qquad \varepsilon^{A'B'C'D'}\Gamma^{D'} = -\Gamma^{A'B'C'}\Gamma_{789(10)}, \tag{A.7}$$

and

$$\Gamma_{789(10)}\epsilon = -\Gamma_{3456}\epsilon, \tag{A.8}$$

which is implied by $\Gamma_{012}\epsilon = \epsilon$ and $\Gamma_{0123456789(10)} = -1$. We also have that

$$\begin{aligned} \delta_\mu \mathcal{L} &= -\frac{i}{2}\partial_\mu \text{Tr}(\bar{\Psi}\Gamma^\mu, \delta_\mu \Psi) - i\mu \text{Tr}(D_\mu X^I, \bar{\Psi}\Gamma_{3456}\Gamma^\mu \Gamma^I \epsilon) \\ &\quad -i\frac{1}{2}\mu \text{Tr}([X^I, X^J, X^K], \bar{\Psi}\Gamma^{IJ}\Gamma_{3456}\Gamma^K \epsilon) \end{aligned} \tag{A.9}$$

and that

$$\delta_\mu \mathcal{L}_{\rm mass} = \mu^2 {\rm Tr}(i\bar{\epsilon}\Gamma^I\Psi, X^I). \tag{A.10}$$

Combining all the pieces together we get

$$\begin{aligned}\tilde{\delta}\tilde{\mathcal{L}} = & -i\frac{1}{6}\mu{\rm Tr}([X^I, X^J, X^K], \bar{\Psi}\Gamma_{3456}\Gamma^{IJK}\epsilon) \\ & -i\frac{1}{2}\mu{\rm Tr}([X^I, X^J, X^K], \bar{\Psi}\Gamma^{IJ}\Gamma_{3456}\Gamma^K\epsilon) \\ & -i\frac{2}{3}\mu{\rm Tr}([X^A, X^B, X^C], \bar{\Psi}\Gamma^{ABC}\Gamma_{3456}\epsilon) \\ & +i\frac{2}{3}\mu{\rm Tr}([X^{A'}, X^{B'}, X^{C'}], \bar{\Psi}\Gamma^{A'B'C'}\Gamma_{3456}\epsilon),\end{aligned} \tag{A.11}$$

where we have omitted the surface term in (A.9). Using the identities

$$\begin{aligned}\left[X^I, X^J, X^K\right]\Gamma_{3456}\Gamma^{IJK} = & -\left[X^A, X^B, X^C\right]\Gamma^{ABC}\Gamma_{3456} + 3\left[X^A, X^B, X^{A'}\right]\Gamma^{ABA'}\Gamma_{3456} \\ & -3\left[X^{A'}, X^{B'}, X^{A}\right]\Gamma^{A'B'A}\Gamma_{3456} + \left[X^{A'}, X^{B'}, X^{C'}\right]\Gamma^{A'B'C'}\Gamma_{3456}\end{aligned} \tag{A.12}$$

$$\begin{aligned}\left[X^I, X^J, X^K\right]\Gamma^{IJ}\Gamma_{3456}\Gamma^{K} = & -\left[X^A, X^B, X^C\right]\Gamma^{ABC}\Gamma_{3456} - \left[X^A, X^B, X^{A'}\right]\Gamma^{ABA'}\Gamma_{3456} \\ & +\left[X^{A'}, X^{B'}, X^{A}\right]\Gamma^{A'B'A}\Gamma_{3456} + \left[X^{A'}, X^{B'}, X^{C'}\right]\Gamma^{A'B'C'}\Gamma_{3456},\end{aligned} \tag{A.13}$$

one can show that the right hand side of (A.11) vanishes. This implies that the the deformed field theory is invariant under sixteen linearly realized supersymmetries.

The proposed non-linearly realized supersymmetry transformations are given by

$$\begin{aligned}\delta_n X^I_a &= 0 \\ \delta_n \Psi &= \exp\left(-\frac{1}{3}\mu\Gamma_{3456}\Gamma_\mu\sigma^\mu\right)T^0\eta \\ \delta_n \tilde{A}_\mu{}^b{}_a &= 0,\end{aligned} \tag{A.14}$$

where now η is an eleven dimensional Majorana spinor subject to the constraint $\Gamma_{012}\eta = -\eta$ and T^0 is a central generator of the 3-algebra. The variation of the Lagrangian (2.6) gives

$$\begin{aligned}\delta_n\tilde{\mathcal{L}} &= i\bar{\Psi}^a\Gamma^\mu(D_\mu\delta_n\Psi)_a + \frac{i}{2}\bar{\Psi}_b\Gamma_{IJ}X^I_cX^J_d\delta_n\Psi_a f^{abcd} + i\mu\bar{\Psi}^a\Gamma_{3456}\delta_n\Psi_a \\ & \quad -\frac{i}{2}\partial_\mu(\bar{\Psi}^a\Gamma^\mu\delta_n\Psi_a) \\ &= i\bar{\Psi}^0\Gamma^\mu\partial_\mu(e^{-\frac{1}{3}\mu\Gamma_{3456}\Gamma_\mu\sigma^\mu})\eta + i\mu\bar{\Psi}^0\Gamma_{3456}e^{-\frac{1}{3}\mu\Gamma_{3456}\Gamma_\mu\sigma^\mu}\eta \\ &= 0,\end{aligned} \tag{A.15}$$

where in the second step we used that $f^{cd0}{}_b = 0$ — T^0 being central — and have ignored a total derivative. Therefore the deformed field theory (2.6) is also invariant under sixteen non-linearly realized supersymmetries.

When the deformed field theory is placed on $R\times T^2$ the three dimensional Poincare symmetry is broken. In this case the theory is invariant under the following transformations:

$$
\begin{aligned}
\delta_n X^I_a &= 0\\
\delta_n \Psi &= \exp\left(-\mu\Gamma_{3456}\Gamma_0\sigma^0\right) T^0\eta\\
\delta_n \tilde{A}_\mu{}^b{}_a &= 0.
\end{aligned}
\tag{A.16}
$$

B. Noether charges and supersymmetry algebra

The charges that generate the symmetry transformations of the deformed field theory on $R\times T^2$ are given by

$$
\begin{aligned}
P^+ &= \int d^2\sigma\\
P^I &= \int d^2\sigma\left(\Pi^I_0\cos(\mu\sigma^0)+\mu X^I_0\sin(\mu\sigma^0)\right)\\
K^I &= \int d^2\sigma\left(\Pi^I_0\frac{\sin(\mu\sigma^0)}{\mu}-X^I_0\cos(\mu\sigma^0)\right)\\
J^{AB} &= -i\int d^2\sigma\left(\mathrm{Tr}(X^A,\Pi^B)-\mathrm{Tr}(X^B,\Pi^A)+\frac{i}{4}\mathrm{Tr}(\bar\Psi,\Gamma^{AB}\Gamma^0\Psi)\right)\\
J^{A'B'} &= -i\int d^2\sigma\left(\mathrm{Tr}(X^{A'},\Pi^{B'})-\mathrm{Tr}(X^{B'},\Pi^{A'})+\frac{i}{4}\mathrm{Tr}(\bar\Psi,\Gamma^{A'B'}\Gamma^0\Psi)\right)\\
Q &= \int d^2\sigma\Big(-\mathrm{Tr}(D_\mu X^I,\Gamma^\mu\Gamma^I\Gamma^0\Psi)-\frac{1}{6}\mathrm{Tr}([X^I,X^J,X^K],\Gamma^{IJK}\Gamma^0\Psi)\\
&\qquad +\mu\Gamma^I\Gamma_{3456}\Gamma^0\mathrm{Tr}(X^I,\Psi)\Big)\\
q &= -i\int d^2\sigma\Gamma^0\exp\left(-\mu\Gamma_{3456}\Gamma_0\sigma^0\right)\Psi_0,
\end{aligned}
\tag{B.1}
$$

where $\int d^2\sigma$ is the integral over the T^2. The Hamiltonian of the theory is given by:

$$
\begin{aligned}
H =\ &\Pi^I_a f^{cdba}A_{0cd}X^I_b+\frac{1}{2}\Pi^I_a\Pi^{aI}+\frac{1}{2}D_iX^I_aD_iX^{aI}\\
&+\frac{i}{2}\bar\Psi^a\Gamma^0\dot\Psi_a-\frac{i}{2}\bar\Psi^a\Gamma^0D_0\Psi_a-\frac{i}{2}\bar\Psi^a\Gamma^iD_i\Psi_a\\
&+\frac{i}{4}\mathrm{Tr}([\bar\Psi\Gamma_{IJ},\Psi,X^I],X^J)+V+\frac{1}{2}\mu^2\mathrm{Tr}(X^I,X^I)-\frac{i}{2}\mu\mathrm{Tr}(\bar\Psi,\Gamma_{3456}\Psi)\\
&+\frac{1}{6}\mu\varepsilon^{ABCD}\mathrm{Tr}([X^A,X^B,X^C],X^D)+\frac{1}{6}\mu\varepsilon^{A'B'C'D'}\mathrm{Tr}([X^{A'},X^{B'},X^{C'}],X^{D'})\\
&+\Lambda^{cd\lambda}\dot A_{cd\lambda}-\frac{1}{2}\varepsilon^{\mu\nu\lambda}(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd}+\frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}).
\end{aligned}
\tag{B.2}
$$

Alternatively, one can write:

$$
\begin{aligned}
H =\ &\Pi^I_a f^{cdba}A_{0cd}X^I_b+\frac{1}{2}\Pi^I_a\Pi^{aI}+\frac{1}{2}D_iX^I_aD_iX^{aI}\\
&+\frac{i}{2}\bar\Psi_a\Gamma^0f^{cdba}A_{0cd}\Psi_b-\frac{i}{2}\bar\Psi^a\Gamma^iD_i\Psi_a
\end{aligned}
$$

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$$
\begin{aligned}
&+\frac{i}{4}\mathrm{Tr}([\bar{\Psi}\Gamma_{IJ},\Psi,X^I],X^J)+V+\frac{1}{2}\mu^2\mathrm{Tr}(X^I,X^I)-\frac{i}{2}\mu\mathrm{Tr}(\bar{\Psi},\Gamma_{3456}\Psi)\\
&+\frac{1}{6}\mu\varepsilon^{ABCD}\mathrm{Tr}([X^A,X^B,X^C],X^D)+\frac{1}{6}\mu\varepsilon^{A'B'C'D'}\mathrm{Tr}([X^{A'},X^{B'},X^{C'}],X^{D'})\\
&-\frac{1}{2}\varepsilon^{\mu i\lambda}(f^{abcd}A_{\mu ab}\partial_i A_{\lambda cd})-\frac{1}{3}\epsilon^{\mu\nu\lambda}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}.
\end{aligned}
\tag{B.3}
$$

where $i=1,2$.

In order to calculate the algebra generated by these charges we need the canonical momenta. Π^I is the momentum density conjugate to X^I and satisfies

$$
[X^I_a(\sigma^i),\Pi^J_b(\sigma'^i)]=i\delta^2(\sigma^i-\sigma'^i)\delta_{ab}\delta^{IJ},
\tag{B.4}
$$

where $i=1,2$ are the spatial coordinates on the membrane and $\Pi^I_a=D_0X^I_a$. For the canonical commutation relation for the spinors, one must use Dirac brackets, which for the case of Majorana spinors results in the following commutation relation:

$$
\{\Psi^\alpha_a(\sigma^i),\Psi^\beta_b(\sigma'^i)\}=-\delta^2(\sigma^i-\sigma'^i)\delta_{ab}\delta^{\alpha\beta}
\tag{B.5}
$$

where α,β are eleven dimensional spinor indices.

To compute the action of the symmetries on the fields, we compute the commutator of the charges with the fields. We get

$$
\begin{aligned}
\left[P^I,X^J\right]&=-i\delta^{IJ}\cos(\mu\sigma^0)T^0\\
\left[K^I,X^J\right]&=-i\delta^{IJ}\frac{\sin(\mu\sigma^0)}{\mu}T^0\\
\left[J^{AB},X^C\right]&=-X^A\delta^{BC}+X^B\delta^{AC}\\
\left[J^{AB},\Psi\right]&=-\frac{1}{2}\Gamma^{AB}\Psi\\
\left[J^{A'B'},X^{C'}\right]&=-X^{A'}\delta^{B'C'}+X^{B'}\delta^{A'C'}\\
\left[J^{A'B'},\Psi\right]&=-\frac{1}{2}\Gamma^{A'B'}\Psi\\
\left[\bar{\epsilon}Q,X^J\right]&=i\bar{\epsilon}\Gamma^I\Psi\\
[\bar{\epsilon}Q,\Psi]&=D_\mu X^I\Gamma^\mu\Gamma^I\epsilon-\frac{1}{6}\left[X^I,X^J,X^K\right]\Gamma^{IJK}\epsilon-\mu\Gamma_{3456}\Gamma^IX^I\epsilon\\
[\bar{\epsilon}Q,A_{abi}]&=i\bar{\epsilon}\Gamma_i\Gamma^IX^I_{[a}\Psi_{b]}\\
[\bar{\eta}q,\Psi]&=\exp\left(-\mu\Gamma_{3456}\Gamma_0\sigma^0\right)T^0\eta.
\end{aligned}
\tag{B.6}
$$

We now show that the Noether charges (B.1) of the deformed field theory (2.6) satisfy the Type IIB plane wave superalgebra. For the even generators we get:

$$
\begin{aligned}
&\left[P^I,H\right]=i\mu^2K^I && \left[K^I,H\right]=-iP^I \quad \left[P^I,K^J\right]=i\delta^{IJ}P^+\\
&\left[P^A,J^{BC}\right]=-\delta^{AB}P^C+\delta^{AC}P^B && \left[P^{A'},J^{B'C'}\right]=-\delta^{A'B'}P^{C'}+\delta^{A'C'}P^{B'}\\
&\left[K^A,J^{BC}\right]=-\delta^{AB}K^C+\delta^{AC}K^B && \left[K^{A'},J^{B'C'}\right]=-\delta^{A'B'}K^{C'}+\delta^{A'C'}K^{B'}
\end{aligned}
$$

$$\left[J^{AB}, J^{CD}\right] = -\delta^{BC} J^{AD} + \delta^{AC} J^{BD} + \delta^{BD} J^{AC} - \delta^{AD} J^{BC}$$
$$\left[J^{A'B'}, J^{C'D'}\right] = -\delta^{B'C'} J^{A'D'} + \delta^{A'C'} J^{B'D'} + \delta^{B'D'} J^{A'C'} - \delta^{A'D'} J^{B'C'}. \tag{B.7}$$

The commutation relations between odd and even generators are:

$$\begin{aligned}
\left[P^I, Q\right] &= -i\mu\Gamma^I\Gamma_{3456} q & \left[K^I, Q\right] &= -i\Gamma^I\Gamma^0 q \\
[H, Q] &= 0 & [H, q] &= -i\Gamma_{3456}\Gamma^0 q \\
\left[J^{AB}, Q\right] &= -\frac{1}{2}\Gamma^{AB} Q & \left[J^{AB}, q\right] &= -\frac{1}{2}\Gamma^{AB} q \\
\left[J^{A'B'}, Q\right] &= -\frac{1}{2}\Gamma^{A'B'} Q & \left[J^{A'B'}, q\right] &= -\frac{1}{2}\Gamma^{A'B'} q.
\end{aligned} \tag{B.8}$$

The anticommutators of the supercharges are:

$$\{q^\alpha, q^\beta\} = i\delta^{\alpha\beta} P^+ \qquad \{q^\alpha, Q^\beta\} = -\frac{i}{2}(\Gamma^I\Gamma^0)^{\alpha\beta} P^I - \mu\frac{i}{2}(\Gamma_{3456}\Gamma^I)^{\alpha\beta} K^I$$
$$\{Q^\alpha, Q^\beta\} = 2H\delta^{\alpha\beta} + i\mu(\Gamma^{AB}\Gamma_{3456}\Gamma^0)^{\alpha\beta} J^{AB} + i\mu(\Gamma^{A'B'}\Gamma_{789(10)}\Gamma^0)^{\alpha\beta} J^{A'B'}. \tag{B.9}$$

This is the superalgebra of the Type IIB plane wave [20] (see also [46] for a useful summary of the superalgebra)

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